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# The Jastrow cluster expansion method for finite nuclei

C S Warke and M R Gunye

Tata Institute of Fundamental Research, Bombay 5, India

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**Abstract.** The binding energies and RMS radii of nuclei in the mass region $16 \leqslant A \leqslant 208$ are calculated in the Jastrow cluster expansion method by including the second-order correction to the energy. The Reid soft-core NN interaction has been employed in the calculations. The binding energies are predicted reasonably well for the nuclei in the mass region $A \sim 90$ whereas they are underestimated in nuclei with $A < 56$ and overestimated in nuclei with $A > 120$. The calculated RMS radii are in good agreement with the experimental data for nuclei with $A \leqslant 56$ but not for heavy nuclei where considerable compression is predicted. The contribution of the three-body cluster correction to the two-body correlation energy is estimated to be at the most 9%, thereby indicating a good convergence of the cluster expansion. Our results are compared with those recently obtained from the Brueckner–Bethe–Goldstone method using the same NN interaction.

## 1. Introduction

A systematic derivation of the cluster expansion of the energy in the Jastrow approach (Jastrow 1955) to the nuclear many-body problem was given by Iwamoto and Yamada (1957). There are very few applications of this method to calculate the binding energies of finite nuclei. Dabrowski (1958a, b) was the first one to investigate the details of this method in connection with the binding energy of $^{16}O$ using a specific central nucleon–nucleon (NN) interaction. Recently there has been considerable interest in finding out a connection between the Brueckner–Bethe–Goldstone (BBG) reaction matrix approach (for a review of this BBG approach, see Day 1967) and the Jastrow cluster expansion (JCE) method. With reasonable and well defined approximations, Day (1971) has shown that the sum of a particular set of diagrams from the perturbation theory gives a many-body wavefunction of the Jastrow form. Attempts (Lassey and Sprung 1971) have also been made to remove some of the approximations made by Day (1971). For a special choice of NN interaction, the nuclear matter binding energy calculated (Wong 1971a) from the JCE method in the independent pair approximation comes out to be much larger than that calculated from the BBG reaction matrix approach. This difference is shown (Wong 1971a) to arise from the absence of the dispersion effects in this simpler form of the JCE method. With the inclusion of dispersion effects and with the special choice of correlation function, many authors (Wong 1971b, Providencia and Shakin 1971, Shakin 1971, Ristig and Clark 1972) have attempted to show how the JCE method for the energy of a many-body system could be related to the BBG reaction matrix method. These suggestions for the special choice of the correlation function implicitly amount to either doing the complicated BBG calculations or making comparatively simpler Jastrow calculations quite difficult. The motivations behind the present work are: (i) to calculate the binding energies and the sizes of nuclei in the mass region

$16 \leqslant A \leqslant 208$ in the JCE approach by employing a realistic NN interaction, since we have not yet noticed such calculations for heavy mass nuclei; (ii) to see whether there is a qualitative agreement between the predictions of the JCE and BBG methods for the nuclear binding energies and sizes; (iii) to find out whether the JCE approach is suitable in the case of realistic NN interactions such as the Reid soft-core potential (Reid 1968); and finally (iv) to study the convergence of the JCE method for the binding energy and RMS radius.

The details of the calculations are given in § 2. The results are presented in § 3 and and the conclusions are summarized in § 4.

## 2. Energy calculation with Jastrow wavefunction

The hamiltonian of the $A$-nucleon system can be written as

$$H = \sum_i T_i + \sum_{i<j} V_{ij}, \tag{1}$$

where $T$ is the kinetic energy operator and $V$ is the NN interaction. We employ the variational wavefunction of the form

$$\Psi = \left[\prod_{i<j} f_{ij}\right] \Phi(1, 2, \ldots, A). \tag{2}$$

Here $\Phi$ is a determinantal wavefunction of $A$ nucleons and $f_{ij}$ is the correlation function. From the nuclear binding energy calculations in the BBG approach, the wound integrals are found to be strongly channel dependent (Grange and Preston 1973, Demos and Banerjee 1972). The wound integral in a particular channel is a measure of the deviation of the correlation function in that channel from unity. This implies that one should employ channel-dependent correlation functions in the Jastrow approach. The effect of the state-dependent correlation functions in the Jastrow approach is discussed by Schafer and Schutte (1972). For a state-dependent correlation function $f_{ij}$, the simple product in equation (2) is not a symmetric function under the exchange of particle coordinates since $f_{ij}$ does not commute with $f_{jk}$. In order to make the total wavefunction $\Psi$ in equation (2) antisymmetric, the product in the square bracket should be symmetrized since the determinant $\Phi$ is an antisymmetric function of $A$ nucleons. The square bracket in equation (2) indicates this symmetrized product of correlation functions. In the calculations reported in this paper, we use the channel ($lSTJ$)-dependent correlation function with one variational parameter $\beta$ in each channel. The form

$$f_{(r)}^{lSTJ} = 1 - e^{-\beta r^2} \tag{3}$$

for the correlation function is chosen in view of the Reid soft-core NN interaction (Reid 1968) employed in the present calculations. The gaussian form is preferred to the exponential form since $df/dr$ vanishes at $r = 0$. The calculations are performed for nuclei which can be described by the closure of either a major shell or a $j$ subshell for both protons and neutrons. In equation (2), the determinantal wavefunction $\Phi$ is constructed from the single-particle orbitals $\phi_\alpha(i)$ where $\alpha$ specifies all the necessary quantum numbers for the orbital. In a variational approach, all the unknown functions $f_{ij}$ and $\phi_\alpha(i)$ should be determined by minimizing the total energy of the system. However, such a general procedure is very laborious in the case of heavy nuclei under investigation. In the absence of correlations ($f_{ij} = 1$), it is found (Gunye and Warke 1970) that the wavefunctions $\Phi$ generated from the single-particle harmonic oscillator wavefunctions

with one variable parameter $b = (\hbar/M\omega)^{1/2}$ provide a good approximation for computing binding energies and RMS radii for the nuclei under consideration here. Hence in the present Jastrow calculations, only the channel parameters $\beta^{lSTJ}$ in the correlation functions $f^{lSTJ}$ and the oscillator parameter $b$ in the determinantal wavefunction $\Phi$ are determined variationally. The Jastrow expansion of the energy $E$ with the correlated wavefunction $\Psi$ can be obtained by following the derivation of Iwamoto and Yamada (1957) and Dabrowski (1958a, b)

$$E = K + E_2 + E_3. \tag{4}$$

The kinetic energy $K$ and the two-body correlation energy $E_2$ can be expressed as

$$K = \sum_i \langle i|T|i\rangle \tag{5}$$

$$E_2 = \tfrac{1}{2}\sum_{ij} \langle ij|W_{12}|ij\rangle_a, \tag{6}$$

where the suffix a indicates the antisymmetrization of the matrix element and

$$W_{12} = f_{12}^2 V_{12} - \frac{\hbar^2}{m} f_{12} \sum_{i=1}^{2} (\tfrac{1}{2}\Delta_i f_{12} + \nabla_i f_{12} \cdot \nabla_i). \tag{7}$$

The non-commutativity of the channel-dependent correlation functions $f_{ij}$, $f_{jk}$ and the interaction operator makes the treatment of the three-body effects very complicated. Since we are interested only in estimating the contribution from the three-body clusters, we neglect the effect of this non-commutativity. In this approximation, the three-body correlation energy can be expressed as

$$\begin{aligned} E_3 = \sum_{ij} (\langle ij|T_1 h_{12}|ij\rangle_a - \tfrac{1}{2}\langle ij|W_{12}|ij\rangle_a \langle ij|h|ij\rangle_a) + \sum_{ijk} (\langle ijk|W_{12}h_{23}|ijk\rangle_a - \langle ij|W_{12}|ij\rangle_a \\ \times \langle jk|h|jk\rangle_a - \langle ij|h|kj\rangle_a \langle k|T|i\rangle + \frac{\hbar^2}{8m}\langle ijk|\nabla_2 h_{12} \cdot \nabla_2 h_{23}|ijk\rangle_a) \\ + \tfrac{1}{4}\sum_{ijkl} (\langle ijkl|W_{12}h_{23}|ijkl\rangle_a - \langle ij|W_{12}|ij\rangle_a \langle kl|h|kl\rangle_a) \end{aligned} \tag{8}$$

with

$$h_{ij} = f_{ij}^2 - 1. \tag{9}$$

The sums in equations (5), (6) and (8) run over the occupied single-particle states in $\Phi$. For spherical nuclei with central NN interactions, the two-body correlation energy $E_2$ can be expressed as

$$E_2 = \tfrac{1}{2}\sum_{nlSTJ} (2T+1)(2J+1) C_{nl,nl}^{STJ} \langle nl|W_{12}^{lSTJ}|nl\rangle. \tag{10}$$

The index $lSTJ$ for $W_{12}$ in equation (10) indicates the channel dependence of the correlation function. The nuclear $C$ coefficients in equation (10) were evaluated previously (Gunye and Warke 1970). In order to simplify the calculations for evaluating the contribution from three-body cluster terms $E_3$, we make the following approximations:

(i) $$\sum_j \langle kj|h|lj\rangle_a \simeq \frac{1}{A}\delta_{kl}\sum_{ij}\langle ij|h|ij\rangle_a$$

(ii) $$\langle ij|h|kl\rangle_a = 0 \qquad \text{for } i, j \neq k, l.$$

The averaging approximation (i) is expected to be good for the short-range correlation

function $h(r_{12})$. The orthogonality of the single-particle wavefunctions will make the matrix element $\langle ij|h|kl\rangle$ quite small for $i, j \neq k, l$ in the case of the short-range function $h$. The explicit calculations are carried out in the case of $^{16}$O to test the validity of these approximations. The value $\Sigma_j\langle kj|h|kj\rangle = -0{\cdot}47$ (for $k = 0s\,\frac{1}{2}$ orbit) is very close to $A^{-1}\Sigma_{ij}\langle ij|h|ij\rangle = -0{\cdot}42$. The value $\langle ij|h|ij\rangle_a = -0{\cdot}01$ for $i = j(0s\,\frac{1}{2}) \neq k = l(1s\,\frac{1}{2})$. The value of $\Sigma_j\langle kj|h|lj\rangle_a$ is exactly zero for $l_k \neq l_l$ or $j_k \neq j_l$. The maximum value of this sum is $-0{\cdot}2$ when $k = 0s\,\frac{1}{2}$ and $l = 1s\,\frac{1}{2}$ orbits. However, the contribution of such terms comes only in one of the three-body cluster correction terms. With the approximations (i) and (ii), $E_3$ can be written as

$$E_3 = -\sum_{ij}\langle ij|W_{12}|ij\rangle_a\left(\langle ij|h|ij\rangle_a + \frac{1}{A}\sum_{kl}\langle kl|h|kl\rangle_a\right) - \frac{\pi\hbar^2}{m}\sum_{\mu}\left(\frac{1}{A}\sum_{ij}\langle ij|O_\mu|ij\rangle_a\sum_{kl}\langle kl|O_\mu^*|kl\rangle_a - \frac{1}{2}\sum_{ij}|\langle ij|O_\mu|ij\rangle|^2\right) \tag{11}$$

with $O_\mu = (\mathrm{d}h/\mathrm{d}r)Y_{1\mu}$. The matrix element $\langle ij|O_\mu|ij\rangle_a$, however, vanishes in view of the parity considerations. Then employing the $C$ coefficients we can finally express $E_3$ in the form

$$E_3 = -\sum_{nlSTJ}(2T+1)(2J+1)C^{STJ}_{nl,nl}\langle nl|W^{lSTJ}|nl\rangle\left(\frac{\Sigma_{n'}C^{STJ}_{n'l,n'l}\langle n'l|h^{lSTJ}|n'l\rangle}{\Sigma_{n'}C^{STJ}_{n'l,n'l}} + \frac{1}{A}\sum_{n'l'S'T'J'}(2T'+1)(2J'+1)C^{S'T'J'}_{n'l',n'l'}\langle n'l'|h^{l'S'T'J'}|n'l'\rangle\right). \tag{12}$$

In reducing the first term in equation (11), we have replaced $\langle ij|h^{lSTJ}|ij\rangle$, the matrix element, by its average value with the corresponding $C$ coefficients for the channel $lSTJ$ as the weighting factors.

The two-body correlation energy $E_2$ in equation (6) indicates that $W_{ij}$ is the effective NN interaction in the model space. In BGG method, the second-order contribution is already included in deriving the effective NN interaction whereas it is not so in the present approach. Consequently one has to explicitly add the second-order (in $W$) contribution to the energy. An additional reason for including the second-order contribution is that the tensor force contributes substantially to the binding energy and, in particular for spherical nuclei, this contribution comes from higher orders in the non-central part of the NN interaction. The usual second-order perturbation correction has only one term whereas in the Jastrow approach, this correction has three terms (Woo 1966). One of these is just like the ordinary second-order correction and the other two terms arise from the non-orthogonality of the complete set of correlated basis functions. In the lowest order of $h$, one of the latter terms is proportional to the off-diagonal matrix element of $h$ and the other to its square. In view of the approximation (ii), we have not evaluated these terms in this paper. Since the proportionality constant in these corrections is the two-particle two-hole energy (evaluated in the Jastrow approach to zeroth order) and its square, this contribution can be quite important. However, due to the complicated nature of these terms, particularly in large nuclear systems, we have no alternative but to omit these corrections in the present paper. We have evaluated only the usual second-order (in $W$) contribution to the energy by following the approach of Bhaduri and Warke (1968). The density-dependent effective interaction is obtained by applying the criterion that it reproduces the correct second-order contribution in

nuclear matter. From this condition, the required effective interaction $V_{\text{eff}}$ is given by

$$\langle \boldsymbol{rR}|V_{\text{eff}}|\boldsymbol{r'R}\rangle = \sum_{k,p,q}{}' \frac{\langle \boldsymbol{rR}|V|\boldsymbol{k}+\boldsymbol{q},\boldsymbol{p}-\boldsymbol{q}\rangle\langle \boldsymbol{k}+\boldsymbol{q},\boldsymbol{p}-\boldsymbol{q}|V|\boldsymbol{r'R}\rangle}{e(\boldsymbol{k},\boldsymbol{p},\boldsymbol{q},U)}. \tag{13}$$

The prime on the summation indicates the sum is to be taken over the regions

$$k, p \leqslant k_{\text{F}} \quad \text{and} \quad |\boldsymbol{k}+\boldsymbol{q}|, |\boldsymbol{p}-\boldsymbol{q}| > k_{\text{F}},$$

where $k_{\text{F}}$ is the Fermi momentum. The energy denominator in equation (13) is

$$e(\boldsymbol{k},\boldsymbol{p},\boldsymbol{q},U) = -\frac{\hbar^2}{m}\left(\boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{p}+\boldsymbol{q})+\frac{2mU}{\hbar^2}\right), \tag{14}$$

where the additional energy for the occupied states is taken as $-U$. After carrying out straightforward algebra and averaging over the directions of $\boldsymbol{Q} = \boldsymbol{q}+\frac{1}{2}(\boldsymbol{k}-\boldsymbol{p})$, we get

$$\langle r|V^l_{\text{eff}}|r'\rangle = 4\pi V^l(r)V^l(r')\sum_{k,p,q}{}' \frac{j_l(Qr)j_l(Qr')}{e(\boldsymbol{k},\boldsymbol{p},\boldsymbol{q},U)}. \tag{15}$$

Here $j_l$ is the spherical Bessel function. In general $V^l = V^l_{\text{c}}+S_{12}V^l_{\text{T}}$ where $V_{\text{c}}(V_{\text{T}})$ is the central (tensor) component of the NN interaction. In order to simplify the expression for $V^l_{\text{eff}}$, we carry out averaging of $1/e$ over $\boldsymbol{k},\boldsymbol{p}$ within the Fermi surface and of $Q$ over the region of integration for fixed $q$. Let us define $P(U, q/2k_{\text{F}})$ and $\bar{Q}(q)$ as follows

$$\int_{\substack{k\leqslant k_{\text{F}}\\ |\boldsymbol{k}+\boldsymbol{q}|>k_{\text{F}}}} \mathrm{d}^3k \int_{\substack{p\leqslant k_{\text{F}}\\ |\boldsymbol{p}-\boldsymbol{q}|>k_{\text{F}}}} \mathrm{d}^3p \frac{1}{e(\boldsymbol{k},\boldsymbol{p},\boldsymbol{q},U)}\left(\int_{k\leqslant k_{\text{F}}}\mathrm{d}^3k\int_{p\leqslant k_{\text{F}}}\mathrm{d}^3p\right)^{-1}$$

$$= -\frac{3}{20}\frac{m}{\hbar^2}\frac{1}{qk_{\text{F}}}P\left(U,\frac{q}{2k_{\text{F}}}\right)$$

$$\bar{Q}^2(q) = \frac{\Sigma'_{k,p}Q^2}{\Sigma'_{k,p}1}.$$

Evaluating these rather involved integrals, we obtain

$$\left(\frac{\bar{Q}}{k_{\text{F}}}\right)^2 = \begin{cases} \dfrac{5\lambda^7-45\lambda^5-53\lambda^4+75\lambda^3+174\lambda^2+125\lambda+63}{40(3-\lambda^2)} & \text{for } \lambda \leqslant 1 \\ \dfrac{40\lambda^2+3}{10} & \text{for } \lambda > 1, \end{cases} \tag{16}$$

where $\lambda = \frac{1}{2}s = q/2k_{\text{F}}$,

$$P(U,x) = x^2[36v^3+23(2v+1)^2-48(v+1)^2+54(v+1)-x^2(8v+3)]$$

$$+4x[5v(2-v)(v+1)^2+5x^2(1-2v^2)-x^4]\ln\left(\frac{v+1-x}{v+1+x}\right)$$

$$+\tfrac{5}{2}x[(2v-3)(2v+1)^2+6x^2(4v^2-1)+x^4]\ln\left(\frac{2v+1-x}{2v+1+x}\right)$$

$$+20vx^4\ln\left(\frac{4(v+1+x)(v+1-x)}{(2v+1+x)(2v+1-x)}\right)$$

$$+4(v+1)^3(v^2-3v+1)\ln\left(\frac{(v+1+x)(v+1-x)}{(v+1)^2}\right)$$

$$+20x^2(1+3v-2v^3)\ln\left(\frac{(2v+1+x)(2v+1-x)}{4(v+1+x)(v+1-x)}\right)$$

$$-40v^3x^2\ln\left(\frac{(2v+1+x)(2v+1-x)}{4v^2}\right) \qquad \text{for } x \leqslant 1$$

$$P(U,x) = 4(v+x)^3+22(v+x)+4[(v+x)^5-5(v+x)^3-5(v+x)^2+1]\ln(v+x+1)$$
$$+4[(v+x)^5-5(v+x)^3+5(v+x)^2-1]\ln(v+x-1)$$
$$+8[5(v+x)^3-(v+x)^5]\ln(v+x) \qquad \text{for } x > 1, \tag{17}$$

where $v = (m/\hbar^2)(U/qk_F)$. For $U = 0$, this expression for $P(U, x)$ reduces to that derived by Euler (1937) and by Levinger *et al* (1960). Using these functions $\bar{Q}(q)$ and $P(U, x)$, the effective interaction takes the final form

$$\langle r|V^l_{\text{eff}}|r'\rangle = -\frac{3}{10\pi}\frac{m}{\hbar^2}k_F V^l(r)V^l(r')\int_0^\infty \mathrm{d}s\, sP(v,\tfrac{1}{2}s)j_l(\bar{Q}r)j_l(\bar{Q}r'). \tag{18}$$

The second-order contribution to energy is incorporated by replacing $W^{lSTJ}$ with $W^{lSTJ}+V^{lSTJ}_{\text{eff}}$ in equation (10) where $V^{lSTJ}_{\text{eff}}$ is obtained from equation (18) after replacing $V^l$ by $W^l$ on the right hand side.

The RMS radii $R_M$ are calculated with centre of mass correction. The final expression for $R_M$ in Jastrow cluster expansion is

$$A^2R^2_M = \sum_{\substack{nlSTJ\\ n'l'}}(2T+1)(2J+1)C^{STJ}_{nl,n'l'}\langle nl|r^2(1+h^{lSTJ})|n'l'\rangle + R^2_3 \tag{19}$$

where the three-particle cluster correction $R^2_3$ to the mean square radius $R^2_M$ is obtained from the corresponding energy expression in equation (12) after replacing $W_{12}$ by $f^2_{12}r^2_{12}$.

## 3. Results and discussion

The binding energies and RMS radii of nuclei in the mass region $16 \leqslant A \leqslant 208$ are calculated by using the Reid soft-core NN interaction (Reid 1968). The variational parameters $\beta^{lSTJ}$ in the correlation function of equation (3) and the oscillator size parameter $b$ are determined by minimizing the energy $K+E_2$ given in equations (5) and (6) respectively. The RMS radius is then calculated from equation (19) using these values for the parameters $b$ and $\beta^{lSTJ}$. The correctons due to second-order and higher clusters are not considered in the minimization process since the exact form of these correction terms is very complicated. The contributions to the energy from these two corrections are computed from equations (12) and (18) by employing the variationally determined values for the parameters $b$ and $\beta^{lSTJ}$. In evaluating the second-order energy contribution from the tensor part of the Reid soft-core interaction, we have used the parameter values $\beta^{lSTJ}$ of the two channels which are coupled by the tensor interaction. The second-order correction to energy is calculated by using the value for the Fermi momentum

$k_F$ obtained by equating $\hbar^2 k_F^2/2m$ to the average of the kinetic energies of the last filled proton and neutron orbits. This gives

$$k_F = \frac{1}{b}(N_p + N_n + 3)^{1/2}$$

where $N_p(N_n)$ is the major oscillator shell quantum number for the last proton (neutron) orbit. It has been verified that the second-order contribution is not sensitive to the slight variation of $k_F$. We, however, find that it is sensitive to the variation in the parameter $U$ in equation (15). For example, $^{90}$Zr, the binding energy per particle comes out to be 8·15, 9·13 and 10·44 MeV for $U = 80$, 60 and 40 MeV respectively. We take $U$ to be the difference of the average first-order energy $K + E_2$ of the nucleus and the average kinetic energy as is done in the self-consistent nuclear matter calculations

$$U = \frac{1}{A}(K + E_2) - \frac{3}{5}\frac{\hbar^2 k_F^2}{2m^*}.$$

The effective mass parameter $m^*$ is found to be $k_F$ dependent in nuclear matter calculations (Grange and Preston 1973). It increases with the decrease of $k_F$. In the present calculations, we have considered the two extreme values $m^* = m$ and $m^* = 0{\cdot}5m$ since for the relevant $k_F$ values corresponding to the nuclei under consideration, $m^*$ lies between these two values.

The results of the calculation for the binding energies and RMS radii for the nuclei in the mass region $16 \leqslant A \leqslant 208$ are presented in table 1. The calculated binding energies shown in table 1 are corrected for Coulomb energy (Khadkikar and Warke 1969) computed from the oscillator wavefunctions $\Phi$ with a variationally determined parameter $b$. In heavy nuclei, the contributions to binding energy coming from the

**Table 1.** The calculated and experimental values of the binding energies per nucleon and the RMS radii of nuclei are tabulated. The binding energies are computed with two extreme values of the effective mass parameter $m^*$. The experimental binding energies are taken from Mattauch *et al* (1965) and experimental RMS radii are from Collard *et al* (1967). The numbers in the brackets are the results of BBG calculations (Davies *et al* 1972).

| Nucleus | Binding energy per nucleon (MeV) | | | RMS radius (fm) | |
|---|---|---|---|---|---|
| | Calculated | | Experiment | Calculated | Experiment |
| | $m^* = m$ | $m^* = 0{\cdot}5m$ | | | |
| $^{16}$O | 1·32 | 0·13 (4·25) | 7·976 | 2·81 (2·62) | 2·75 |
| $^{28}$Si | 3·08 | 1·40 | 8·448 | 3·07 | 3·06 |
| $^{32}$S | 3·38 | 1·65 | 8·493 | 3·19 | 3·22 |
| $^{40}$Ca | 5·29 | 3·18 (4·46) | 8·552 | 3·30 (3·18) | 3·45 |
| $^{48}$Ca | 7·52 | 5·51 | 8·667 | 3·30 | 3·44 |
| $^{56}$Ni | 7·11 | 5·50 | 8·643 | 3·84 | 3·84 |
| $^{72}$Ge | 9·81 | 7·84 | 8·732 | 3·75 | 4·02 |
| $^{88}$Sr | 11·02 | 8·81 | 8·732 | 3·85 | 4·10 |
| $^{90}$Zr | 10·96 | 8·73 | 8·710 | 3·88 | 4·30 |
| $^{96}$Zr | 11·57 | 9·41 | 8·635 | 3·93 | 4·32 |
| $^{120}$Sn | 11·94 | 9·68 | 8·500 | 4·13 | 4·60 |
| $^{140}$Ce | 13·77 | 11·57 | 8·377 | 4·34 | 4·89 |
| $^{146}$Gd | 13·75 | 11·50 | 8·250 | 4·41 | 4·94 |
| $^{208}$Pb | 14·45 | 12·26 (3·46) | 7·867 | 4·68 (4·73) | 5·50 |

second-order and three-body cluster terms are approximately 20% and 9%, respectively, of the first-order potential energy $E_2$ in equation (6). In the light nuclei, however, these contributions are approximately 60% and 1% respectively. The details of the contributions to the binding energy coming from the two-body and three-body clusters and the second-order corrections are displayed in table 2. The total contribution of the correlations to the RMS radii is found to be approximately 6% of the bare RMS calculated with the oscillator wavefunctions.

**Table 2.** The calculated contributions to the binding energy (in MeV) from the two-body correlation energy $V_1$, the second-order correction $V_2$ obtained with two extreme values of the effective mass parameter $m^*$ and the three-body cluster correction $E_3$ are presented.

| | | $V_2$ | | |
|---|---|---|---|---|
| Nucleus | $V_1$ | $m^* = m$ | $m^* = 0{\cdot}5m$ | $E_3$ |
| $^{16}$O | 231·40 | 137·89 | 114·65 | 1·16 |
| $^{28}$Si | 548·07 | 261·04 | 213·88 | 1·54 |
| $^{32}$S | 642·41 | 310·65 | 255·35 | 1·88 |
| $^{40}$Ca | 1012·66 | 472·51 | 387·71 | 46·99 |
| $^{48}$Ca | 1200·01 | 520·70 | 424·35 | 27·53 |
| $^{72}$Ge | 1968·21 | 858·71 | 716·73 | 76·59 |
| $^{88}$Sr | 2643·61 | 1062·39 | 868·56 | 159·32 |
| $^{90}$Zr | 2757·43 | 1105·48 | 904·54 | 183·47 |
| $^{96}$Zr | 2832·76 | 1158·54 | 950·53 | 145·42 |
| $^{120}$Sn | 3960·32 | 1468·51 | 1197·39 | 323·44 |
| $^{140}$Ce | 4303·97 | 1705·53 | 1397·83 | 227·99 |
| $^{146}$Gd | 4632·32 | 1834·44 | 1506·05 | 274·23 |
| $^{208}$Pb | 7328·23 | 1803·05 | 1347·62 | 658·16 |

The computed binding energies are in reasonable agreement with the corresponding experimental values for nuclei in the mass region $A \sim 90$. As compared to the experimental binding energies, the computed values are quite small in the light nuclei whereas they are large in the heavy nuclei. The calculated RMS radii are in good agreement with the experimental values for light nuclei ($A \leqslant 56$) whereas the heavy nuclei show a tendency of collapsing as can be seen from their small sizes and large binding energies in table 1.

The channel dependence of the correlation functions is displayed in table 3. The values of the channel parameter $\beta^{lSTJ}$ and the average correlation volume $\int [(f^{lSTJ})^2 - 1]\, d\tau$ for the important $^1S_0$ and $^3S_1$ channels are insensitive to the variation of the mass number $A$ as seen from table 3. This trend can also be seen from the results obtained in the BBG calculations of binding energies in $^{16}$O and nuclear matter for the same Reid soft-core NN interaction. The wound integrals (which are the measures of the correlation volumes in the Jastrow method) in $^1S_0$ and $^3S_1$ channels are 0·028 and 0·11 in nuclear matter (Grange and Preston 1973) whereas they are 0·026 and 0·14 in $^{16}$O (Demos and Banerjee 1972). It can be seen from table 3 that the correlation volumes for these two channels are of the same order of magnitude as the corresponding wound integrals in BBG calculations (Grange and Preston 1973, Demos and Banerjee 1972). The present results in table 3 also indicate that the correlation volumes of $^1P_1$ and $^3P_1$ channels are larger than the corresponding $^3P_0$ and $^3P_2$ channels and they are very small in the D channels. A similar trend is noticed in the wound integrals of these channels in the nuclear matter calculations carried out in the BBG approach (Grange and Preston 1973).

**Table 3.** The channel parameters $\beta^{lSTJ}$ and the correlation volumes for various nuclei are displayed. In each nucleus, the numbers in the first and second row indicate the values of $\beta^{lSTJ}$ and the correlation volume respectively. The tabulated values correspond to the minimum of the first-order energy as stated in the text.

| Nucleus | $^1S_0$ | $^3S_1$ | $^1P_1$ | $^3P_0$ | $^3P_1$ | $^3P_2$ | $^1D_2$ | $^3D_1$ | $^3D_2$ |
|---|---|---|---|---|---|---|---|---|---|
| $^{16}O$ | 3·19 | 1·20 | 0·48 | 1·27 | 0·57 | 3·10 | 1·18 | 1·18 | 1·18 |
| | 0·018 | 0·068 | 0·057 | 0·008 | 0·042 | 0·001 | 0·001 | 0·001 | 0·001 |
| $^{40}Ca$ | 3·10 | 1·20 | 0·50 | 1·18 | 0·39 | 6·07 | 1·96 | 1·18 | 2·23 |
| | 0·021 | 0·074 | 0·062 | 0·012 | 0·092 | 0·000 | 0·000 | 0·001 | 0·000 |
| $^{48}Ca$ | 3·10 | 1·20 | 0·50 | 1·18 | 0·50 | 6·50 | 2·05 | 1·18 | 2·23 |
| | 0·020 | 0·069 | 0·057 | 0·010 | 0·057 | 0·000 | 0·000 | 0·001 | 0·000 |
| $^{90}Zr$ | 3·10 | 1·20 | 0·45 | 1·20 | 0·41 | 8·16 | 2·75 | 1·27 | 3·10 |
| | 0·017 | 0·061 | 0·059 | 0·009 | 0·068 | 0·000 | 0·000 | 0·001 | 0·000 |
| $^{96}Zr$ | 3·10 | 1·20 | 0·50 | 1·20 | 0·43 | 8·25 | 2·84 | 1·27 | 3·10 |
| | 0·016 | 0·057 | 0·045 | 0·008 | 0·058 | 0·000 | 0·000 | 0·001 | 0·000 |
| $^{120}Sn$ | 3·12 | 1·14 | 0·45 | 1·20 | 0·39 | 8·94 | 3·10 | 1·27 | 3·53 |
| | 0·016 | 0·061 | 0·055 | 0·008 | 0·068 | 0·000 | 0·000 | 0·001 | 0·000 |
| $^{140}Ce$ | 3·10 | 1·20 | 0·50 | 1·21 | 0·45 | 9·12 | 3·19 | 1·35 | 3·53 |
| | 0·014 | 0·051 | 0·039 | 0·007 | 0·047 | 0·000 | 0·000 | 0·001 | 0·000 |
| $^{146}Gd$ | 3·10 | 1·20 | 0·50 | 1·21 | 0·45 | 9·12 | 3·19 | 1·35 | 3·62 |
| | 0·014 | 0·051 | 0·039 | 0·007 | 0·047 | 0·000 | 0·000 | 0·001 | 0·000 |
| $^{208}Pb$ | 3·12 | 1·14 | 0·45 | 1·20 | 0·41 | 10·0 | 3·62 | 1·35 | 4·28 |
| | 0·014 | 0·051 | 0·045 | 0·007 | 0·051 | 0·000 | 0·000 | 0·000 | 0·000 |

## 4. Conclusions

We have calculated the binding energies and sizes of nuclei in the mass region $16 \leqslant A \leqslant 208$ in the JCE approach by employing a Reid soft-core NN interaction (Reid 1968). The computed binding energies are in fair agreement with the experimental values in the mass region $A \sim 90$ whereas they are small in lighter nuclei and large in heavier nuclei as compared to the corresponding experimental values. In this connection we point out two $A$-dependent effects in the evaluation of the second-order contribution to the binding energy. The first is related to the use of the effective mass $m^*$ which should increase in light nuclei (small $k_F$) and decrease in heavy nuclei (large $k_F$) in accordance with the $k_F$ dependence of $m^*$ in nuclear matter calculations (Grange and Preston 1973). The other effect is related to the neglected terms in the second-order correction in the Jastrow approach (Woo 1966). This contribution of the neglected terms has the opposite sign and its $A$ dependence is similar to that of the leading term included in the present calculations. Both these effects are in the right direction to improve the agreement between the calculated and experimental binding energies. The computed small binding energies in light nuclei ($A < 40$) are probably due to the known fact (Iwamoto and Yamada 1957) that the Jastrow expansion becomes poorer for the larger ratio of the correlation volume to the nuclear volume. The calculated sizes are in good agreement with the experimental data for nuclei with $A \leqslant 56$ whereas they are predicted to be quite small for the heavy nuclei. The agreement in the RMS radii can be improved by incorporating the important effect of tensor force on the nuclear saturation properties. The RMS radii obtained in the BBG calculations (Davies *et al* 1972) performed with the same NN interaction are nearly the same as in our calculations though ours are in slightly better agreement with the experimental data. The binding energies computed in the two

methods differ substantially in the case of $^{16}O$ and $^{208}Pb$ whereas they are comparable in the case of $^{40}Ca$. The BBG calculations (Davies *et al* 1972) yield too low a binding energy in $^{208}Pb$.

It should be stressed here that there exists a striking similarity between the wound integrals for various channels in the BBG method and the corresponding correlation volumes in the Jastrow approach suggesting the use of state-dependent correlation functions in the latter approach. The present calculations show that the contribution of the three-body cluster correction to the two-body correlation energy is at the most 9%, thereby indicating a good convergence of the cluster expansion.

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